Modeling Decisions - 2

Decision Supports Systems 2017/18, Lecture 03

Marko Tkalčič

Alpen-Adria-Universität Klagenfurt

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Texaco vs Pennzoil

In early 1984, Pennzoil and Getty Oil agreed to the terms of a merger. But before any formal documents could be signed, Texaco offered Getty a substantially better price, and Gordon Getty, who controlled most of the Getty stock, reneged on the Pennzoil deal and sold to Texaco. Naturally, Pennzoil felt as if it had been dealt with unfairly and immediately filed a lawsuit against Texaco alleging that Texaco had interfered illegally in the Pennzoil-Getty negotiations. Pennzoil won the case; in late 1985, it was awarded $11.1 billion, the largest judgment ever in the United States at that time. A Texas appeals court reduced the judgment by $2 billion, but interest and penalties drove the total back up to $10.3 billion. James Kinnear, Texaco's chief executive officer, had said that Texaco would file for bankruptcy if Pennzoil obtained court permission to secure the judgment by filing liens against Texaco's assets. Furthermore, Kinnear had promised to fight the case all the way to the U.S. Supreme Court if necessary, arguing in part that Pennzoil had not followed Security and Exchange Commission regulations in its negotiations with Getty. In April 1987, just before Pennzoil began to file the liens, Texaco offered to pay Pennzoil $2 billion to settle the entire case. Hugh Liedtke, chairman of Pennzoil, indicated that his advisors were telling him that a settlement between $3 and $5 billion would be fair.

Texaco vs. Pennzoil

- Pennzoil offers Getty oil to buy them (merge)

What should Liedke do?

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- "advisors say $3-$5 bn would be fair"

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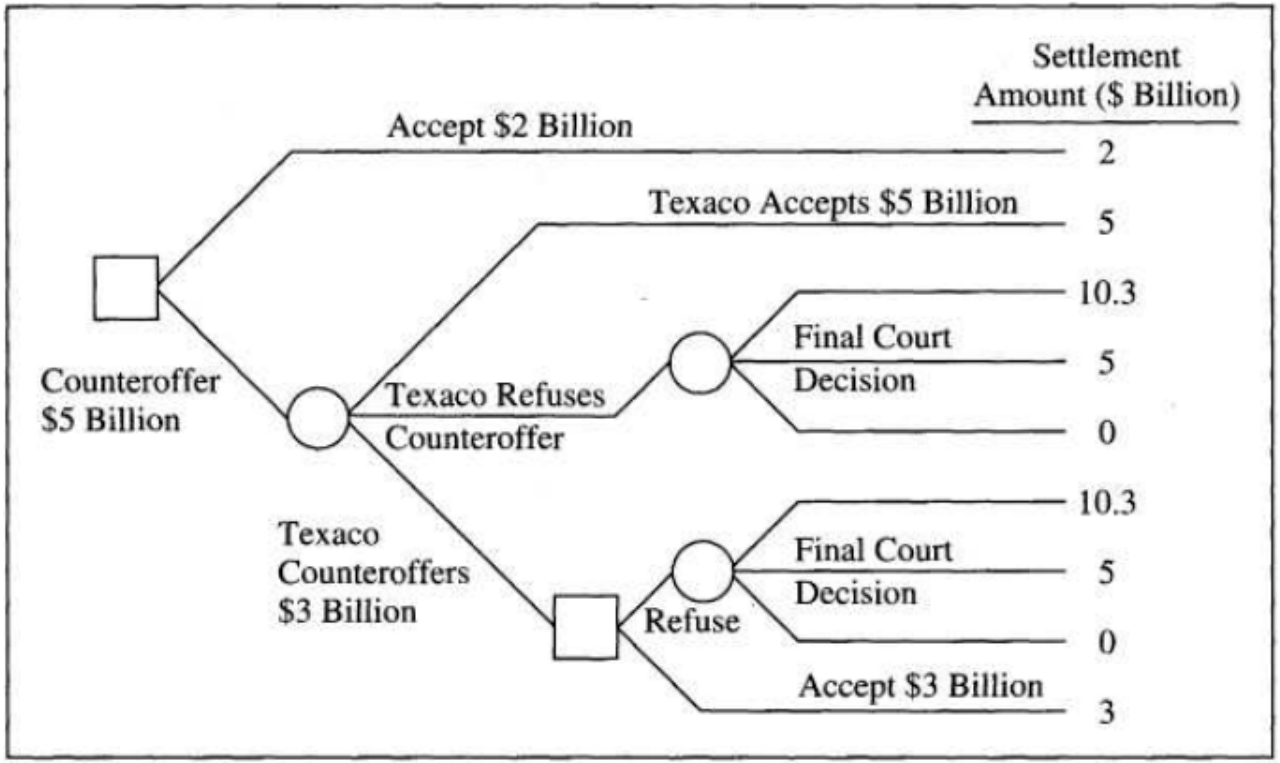
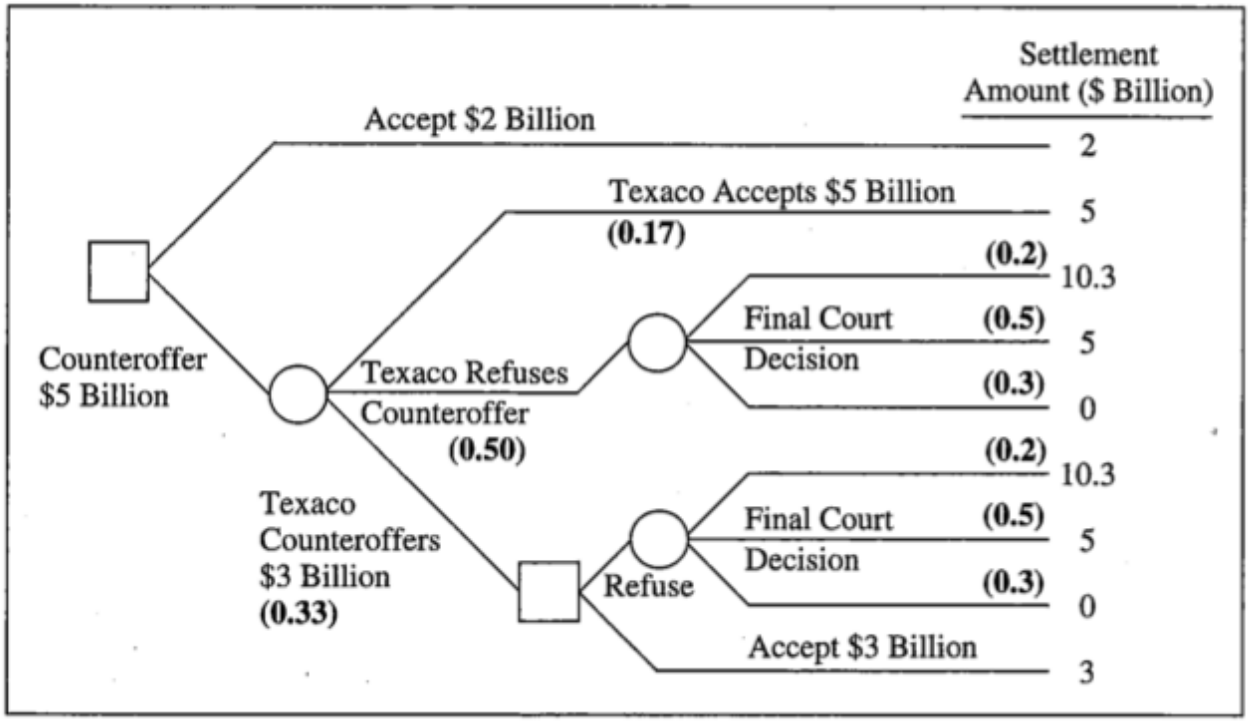
Decision tree

Figure 4.1
Hugh Liedtke's decision in the Texaco-Pennzoil affair.



Decision Tree with Chances

Figure 4.2
Hugh Liedtke's decision tree with chances (probabilities) included.



- we can assign probabilities to chance outcomes ($p \in [0, 1]$)

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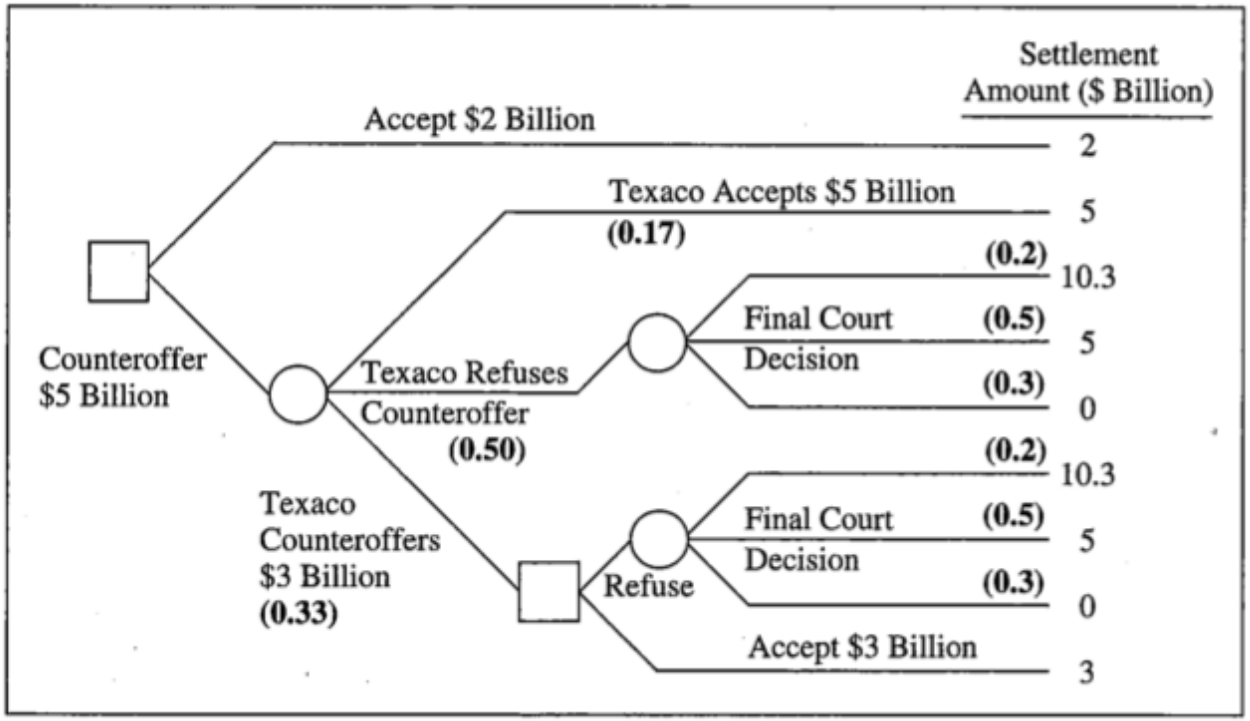


- we can assign probabilities to chance outcomes ($p \in [0, 1]$)
- how to choose between many risky options?

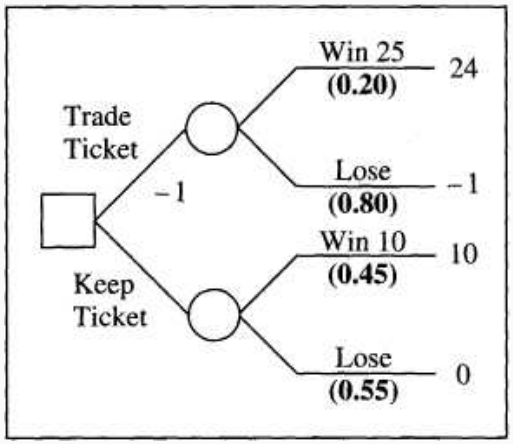
Decision Trees and Expected Monetary Value (EMV)

- how to choose between many risky options?
- case: double risk dilemma
  - you have a lottery ticket:
    - chance: 45%
    - reward: 10 EUR
  - your friend has a lottery ticket
    - chance: 20%
    - reward: 25 EUR
  - decision:
    - trade ticket with friend + 1 EUR
    - keep ticket

Decision Trees and Expected Monetary Value (EMV)

- how to choose between many risky (uncertain) options?
  - **pick the option with the highest *expected monetary value (EMV)***
    - replace the chance nodes by their EMV

Decision Trees and Expected Monetary Value (EMV)

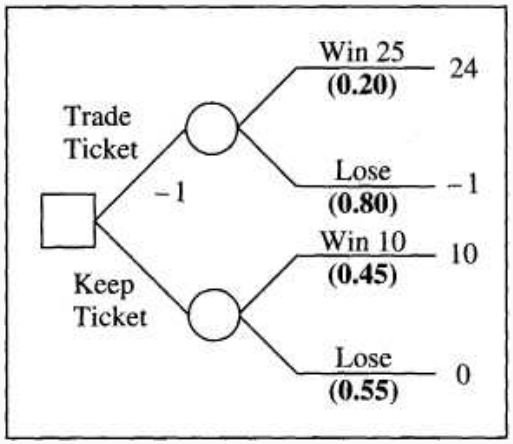


- step 1: calculate/assign the *net values* for the chain of events and decision
- step 2: calculate the EMV for the chance node
  - EMV is the weighted sum of all possible outcomes

$$EMV(ChanceNode) = \sum_{n=1..N} p_n \cdot v_n$$

where N is the number of outcomes, p_n is the probability of outcome n and v_n is the value of outcome n

Decision Trees and Expected Monetary Value (EMV)



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  - EMV is the weighted sum of all possible outcomes

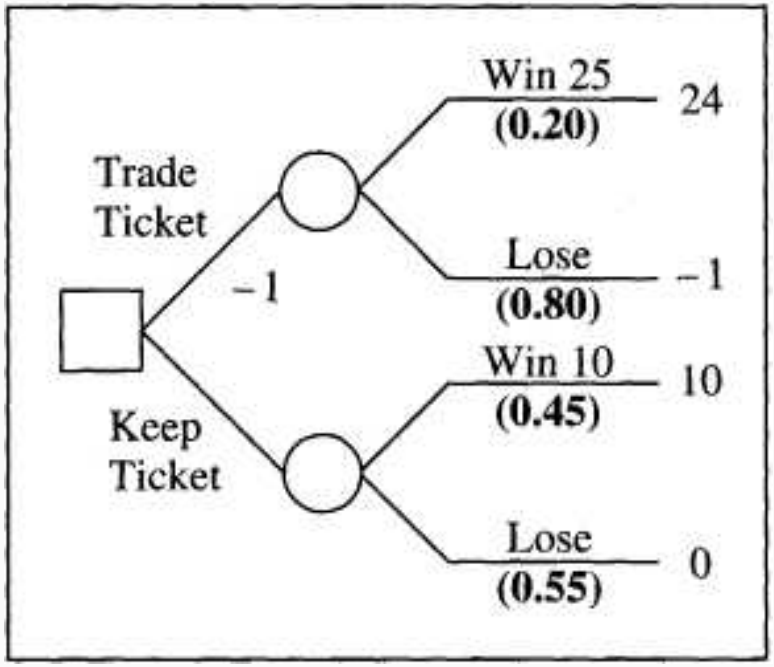
$$EMV(ChanceNode) = \sum_{n=1..N} p_n \cdot v_n$$

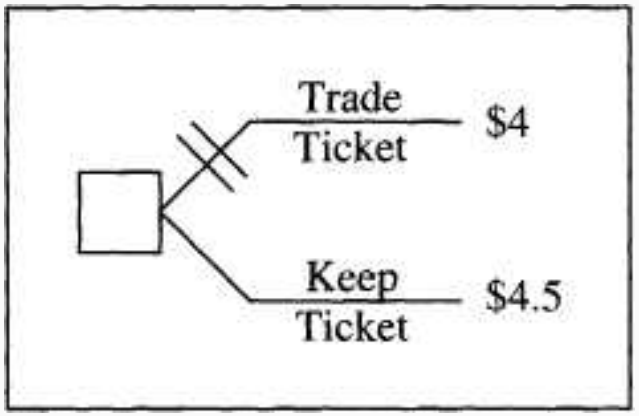
where N is the number of outcomes, p_n is the probability of outcome n and v_n is the value of outcome n

$$EMV(TradeTicket) = 0.2 \cdot (25 - 1) + 0.8 \cdot (-1) = 4$$

$$EMV(KeepTicket) = 0.45 \cdot 10 + 0.55 \cdot 0 = 4.5$$

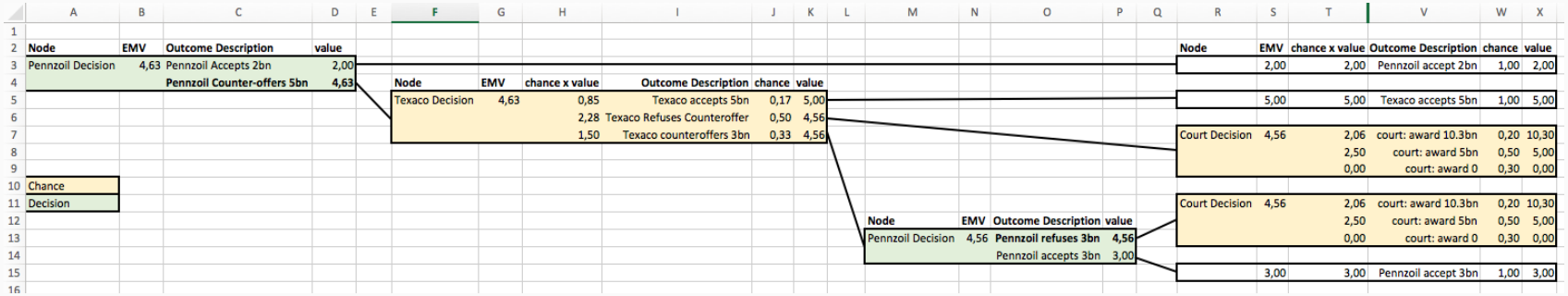
Decision Trees and Expected Monetary Value (EMV)





- we pick the branch with the highest EMV

Example with Spreadsheet



Node	EMV	Outcome Description	value
Pennzoil Decision	4,63	Pennzoil Accepts 2bn	2,00
		Pennzoil Counter-offers 5bn	**4,63**

Node	EMV	chance x value	Outcome Description	chance	value
Texaco Decision	4,63	0,85	Texaco accepts 5bn	0,17	5,00
		2,28	Texaco Refuses Counteroffer	0,50	4,56
		1,50	Texaco counteroffers 3bn	0,33	4,56

Node	EMV	Outcome Description	value
Pennzoil Decision	4,56	**Pennzoil refuses 3bn**	**4,56**
		Pennzoil accepts 3bn	3,00

Node	EMV	chance x value	Outcome Description	chance	value
	2,00	2,00	Pennzoil accept 2bn	1,00	2,00
	5,00	5,00	Texaco accepts 5bn	1,00	5,00
Court Decision	4,56	2,06	court: award 10.3bn	0,20	10,30
		2,50	court: award 5bn	0,50	5,00
		0,00	court: award 0	0,30	0,00
Court Decision	4,56	2,06	court: award 10.3bn	0,20	10,30
		2,50	court: award 5bn	0,50	5,00
		0,00	court: award 0	0,30	0,00
	3,00	3,00	Pennzoil accept 3bn	1,00	3,00

Chance
Decision

[tree-pennzoil-texaco.xlsx]

Risk profile

- In the example: refusing to accept the 3bn can lead to anything between 0 and 10bn
- EMV is not always the best way for assessing the value of a decision alternative

Risk profile

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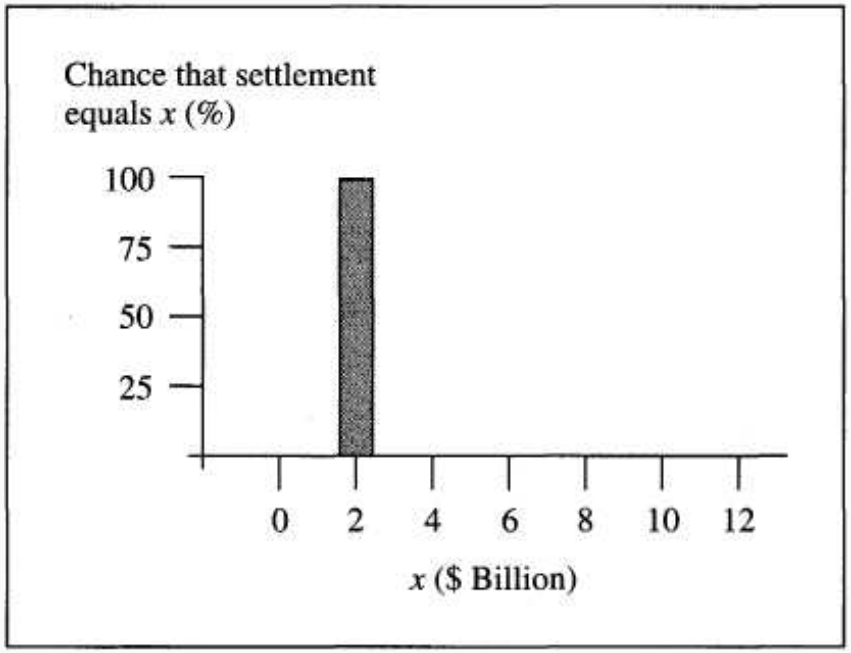
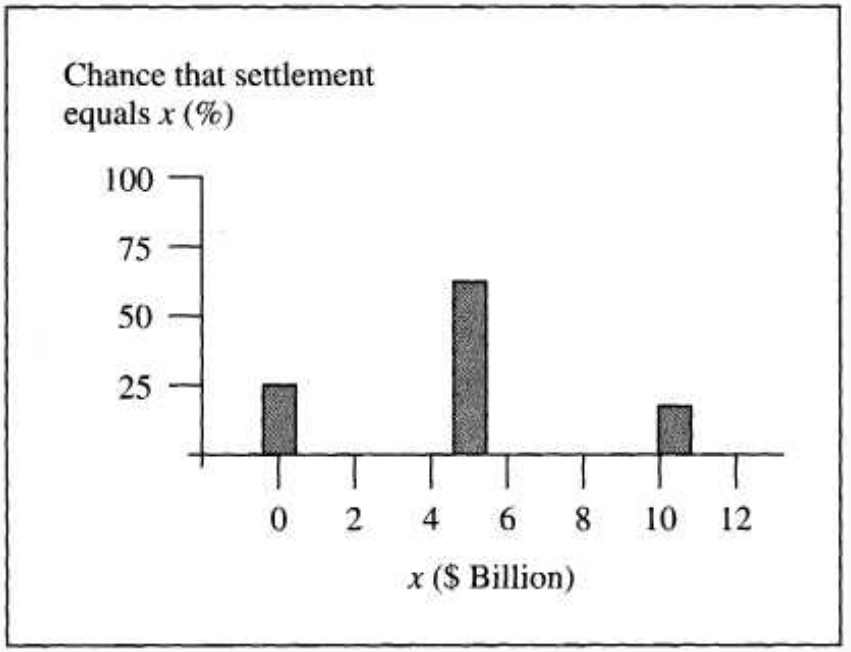
- EMV is not always the best way for assessing the value of a decision alternative
- a **risk profile** is a graph showing the chances associated with possible outcomes of a decision alternative



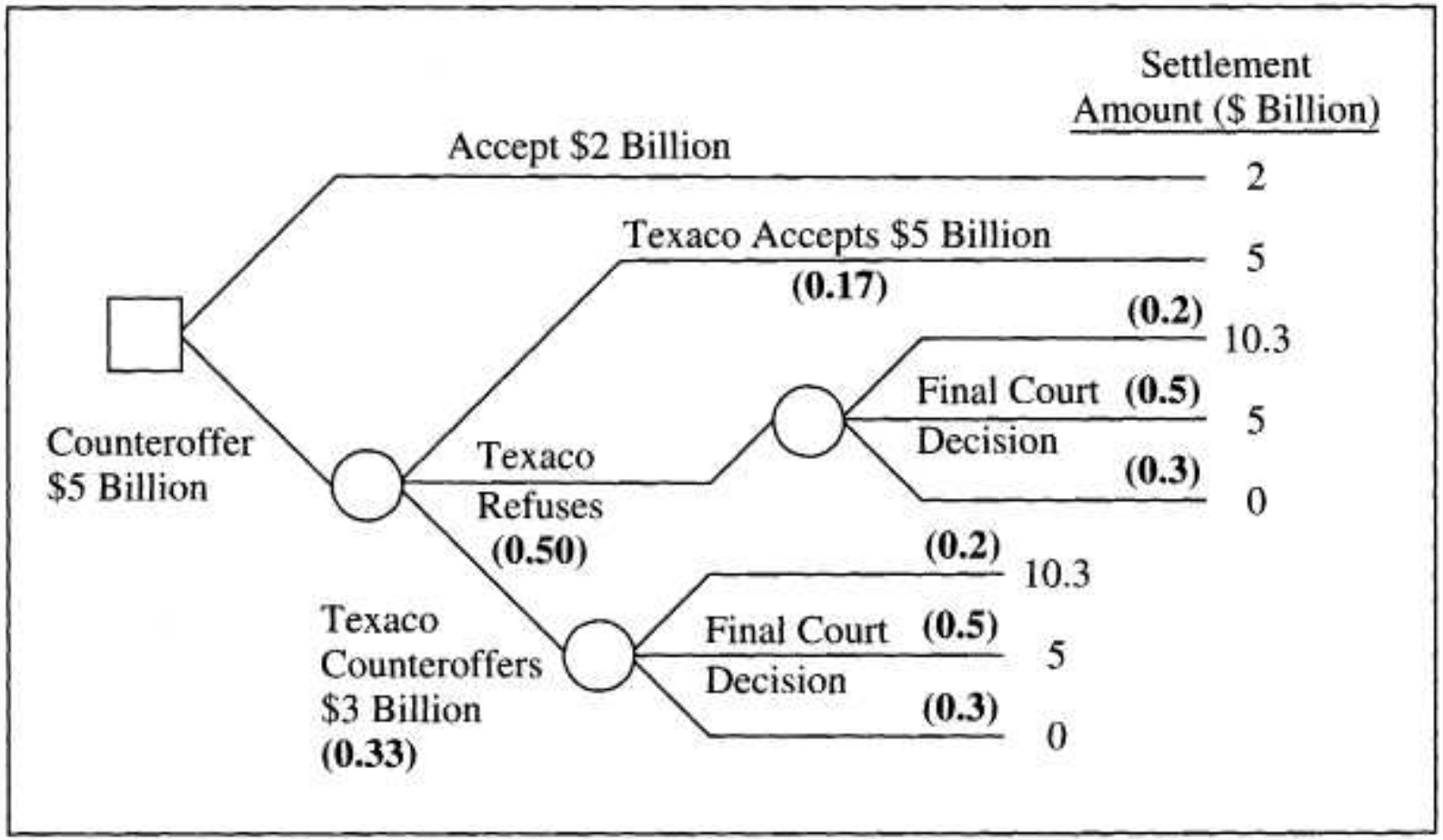
"accept 2bn" alternative



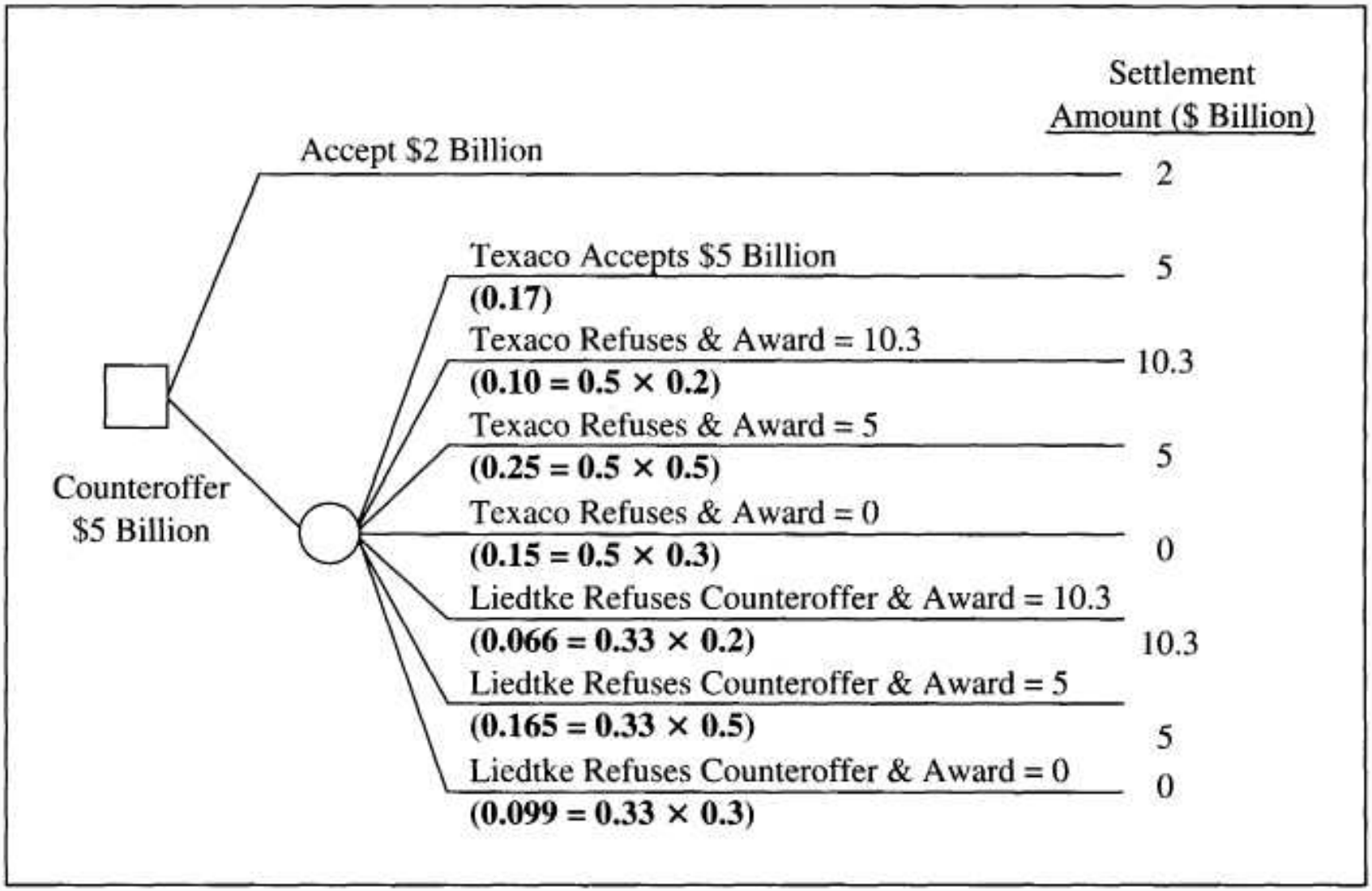
"counter-offer 5bn, refuse Texaco counteroffer" alternative

How did we construct this profile?

Risk profile - collapsing the decision tree



Risk profile - collapsing the decision tree



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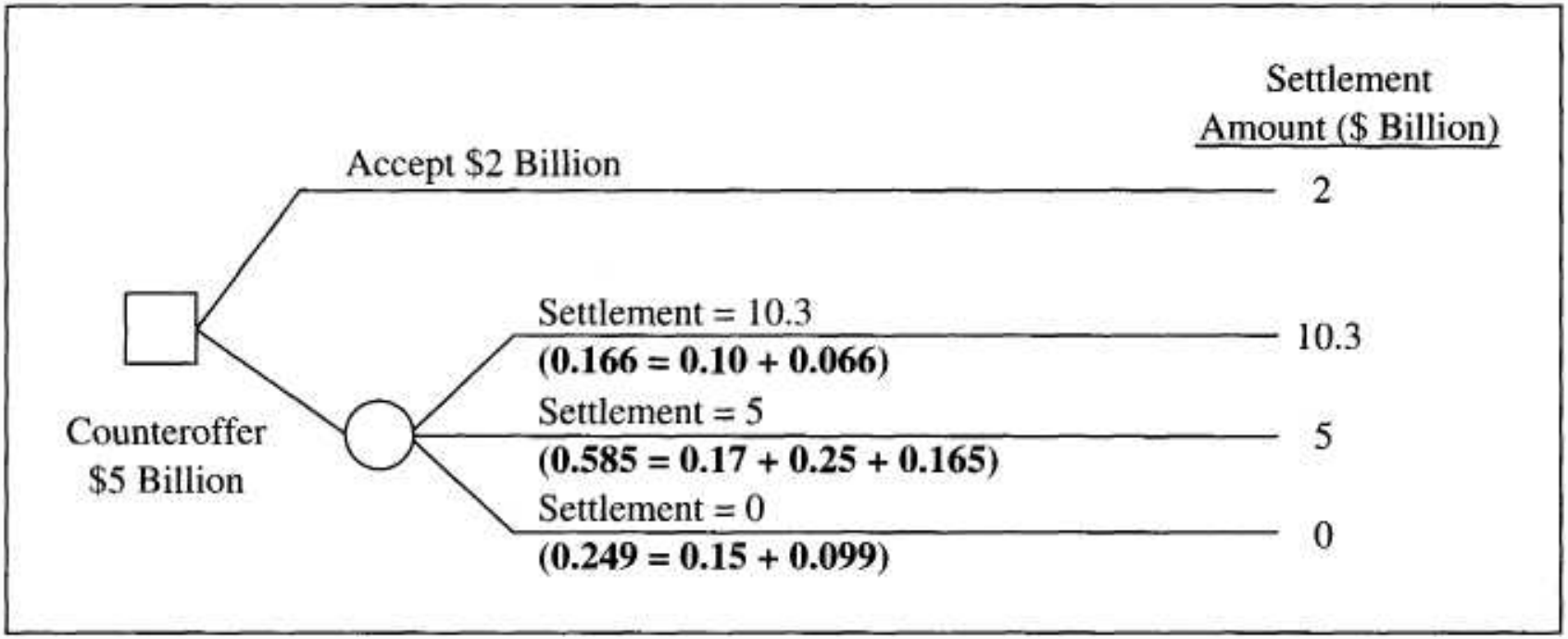
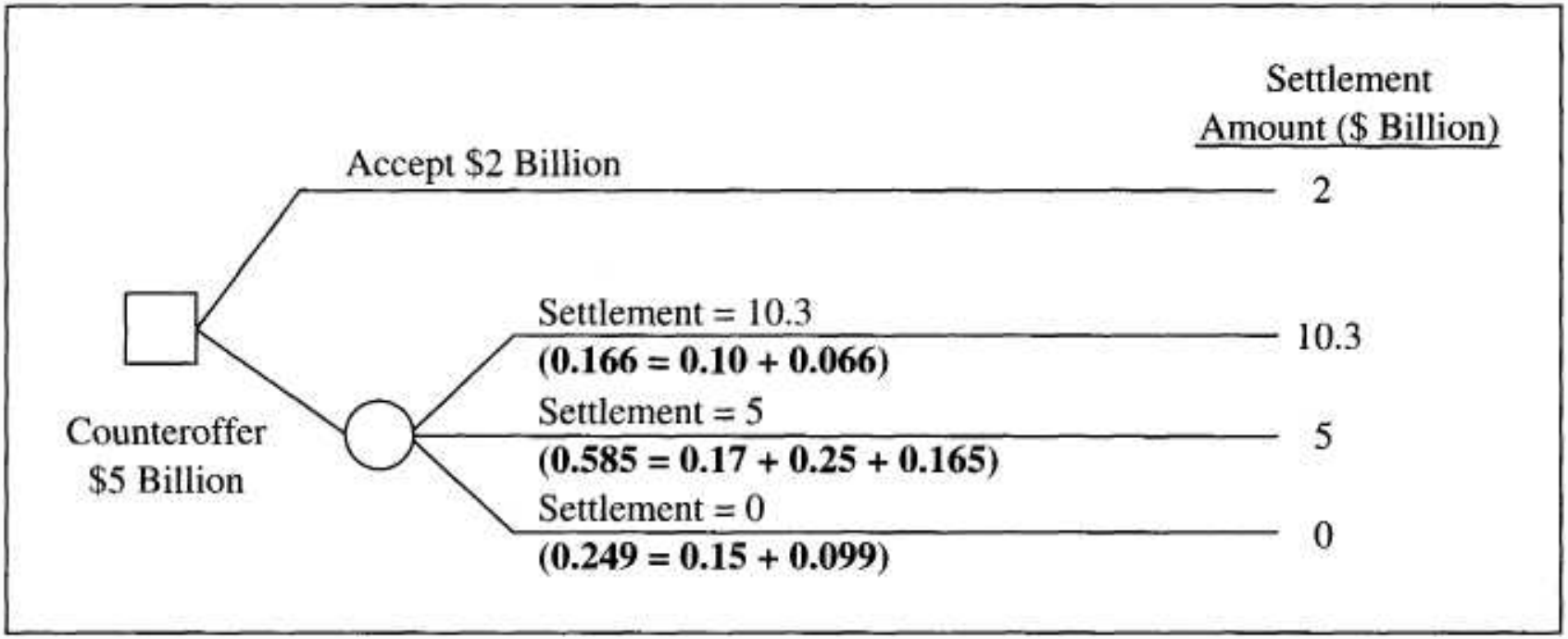
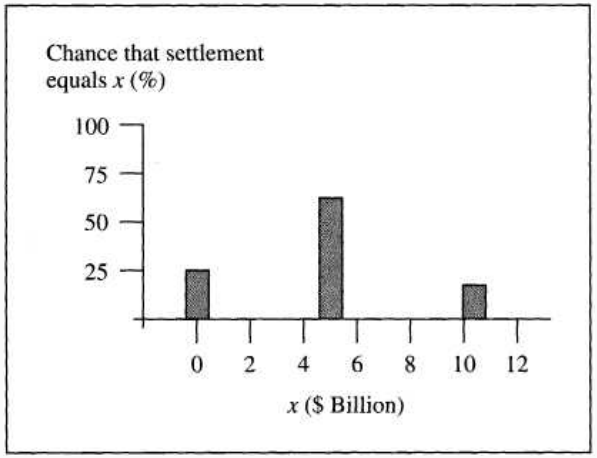


Risk profile - collapsing the decision tree



The EMV (=4.63) can be replaced by the risk profile, which provides more information.



Quiz

Build the risk profile for the "Counteroffer 5bn, Accept 3bn" strategy.

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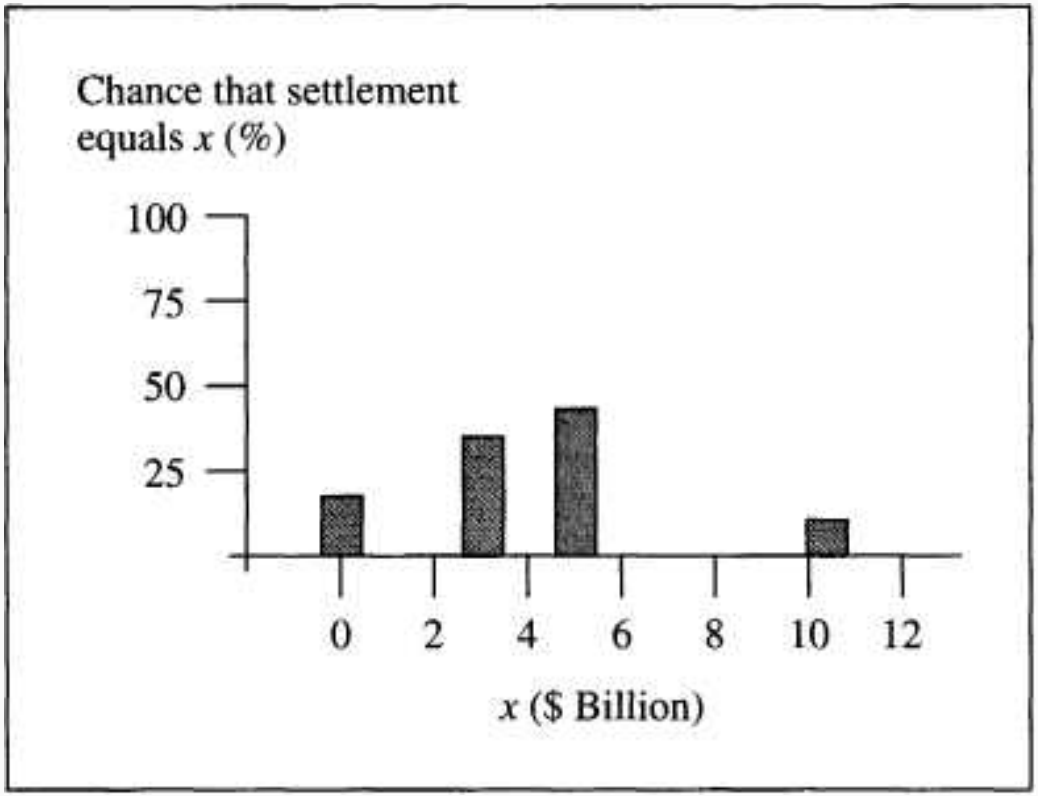

Multiple Objectives and Trade-offs

- The summer-job decision requires Sam to make an explicit trade-off between the objectives of maximizing fun and maximizing salary.
- How can Sam make this trade-off?

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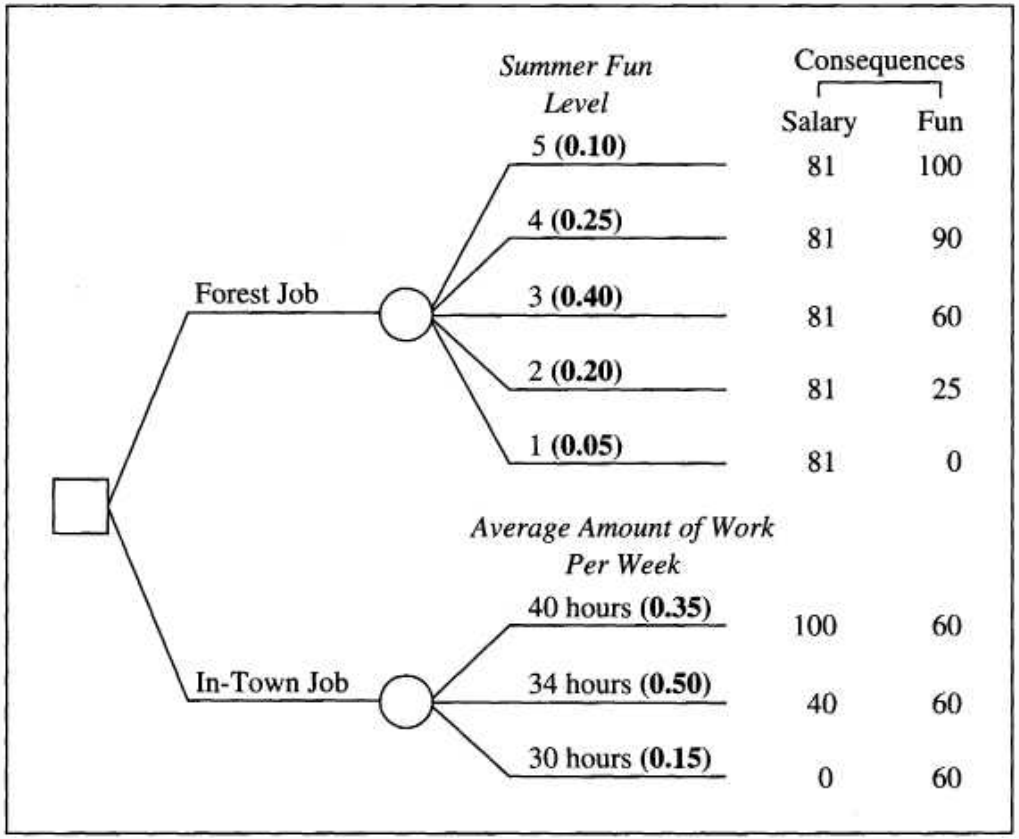


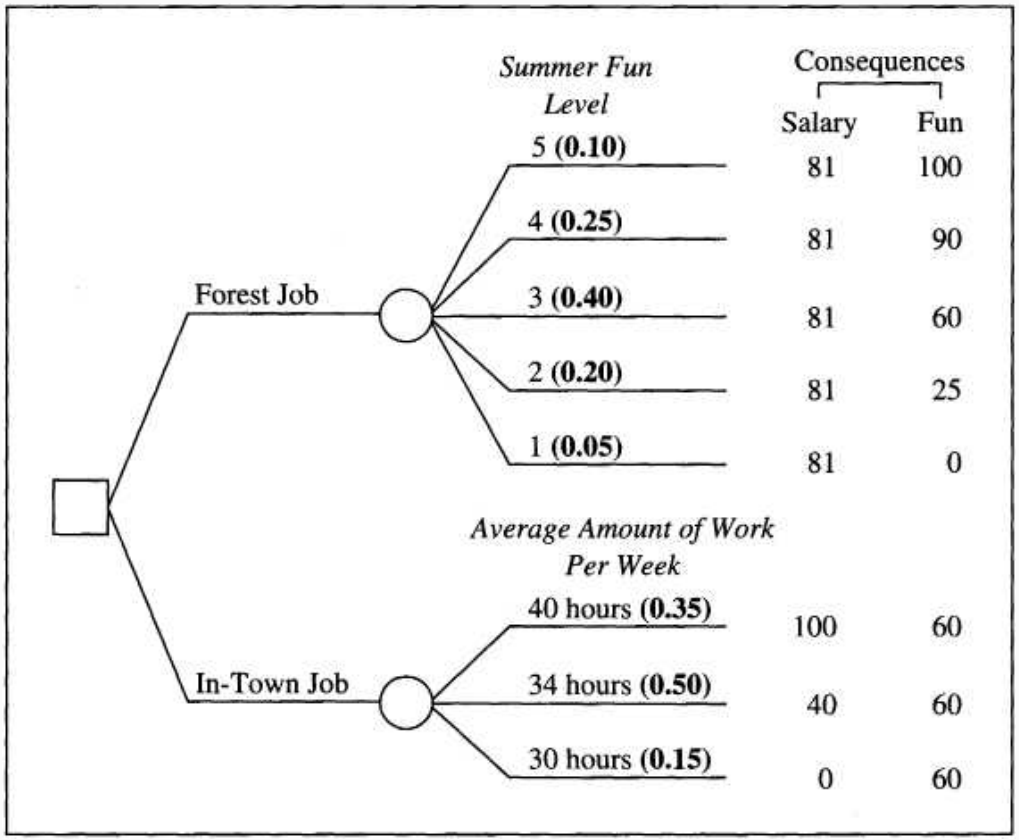
Multiple Objectives and Trade-offs

- trade-offs -> merge the multiple objectives into a single utility using weights

$$u(outcome) = \sum w_i \cdot v_i(outcome)$$

- w_i = weight (importance) of objective i
- v_i = value of the outcome through objective i
- all values should be on the same scale, e.g. [0..1]

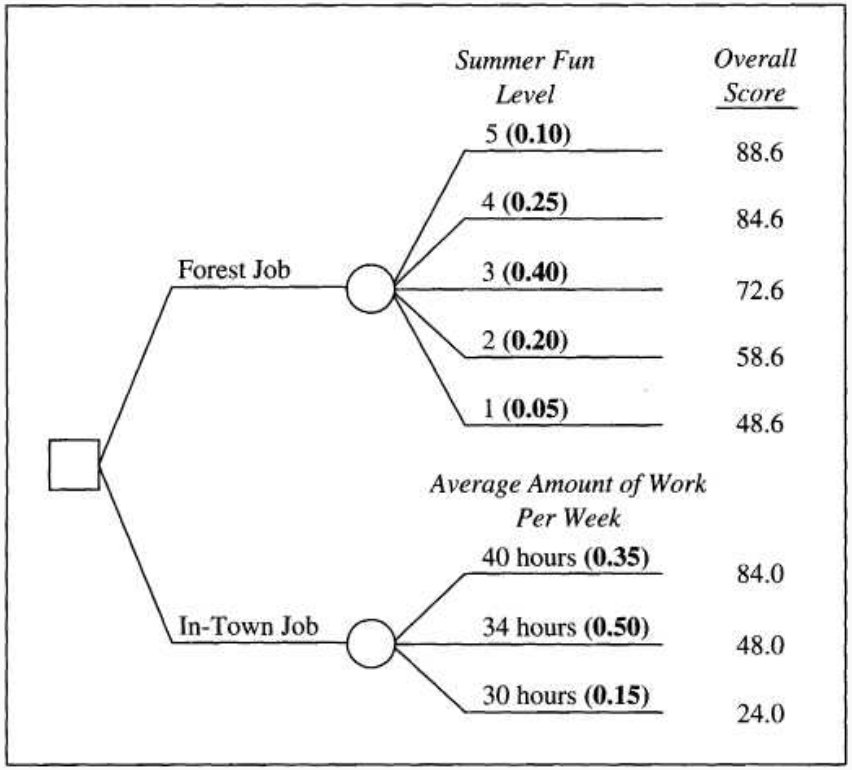
Multiple Objectives and Trade-offs



- salary weight: $w_1 = 0.6$
- fun weight: $w_2 = 0.4$

$$u(outcome) = 0.6 \cdot salary(outcome) + 0.4 \cdot fun(outcome)$$

Multiple Objectives and Trade-offs

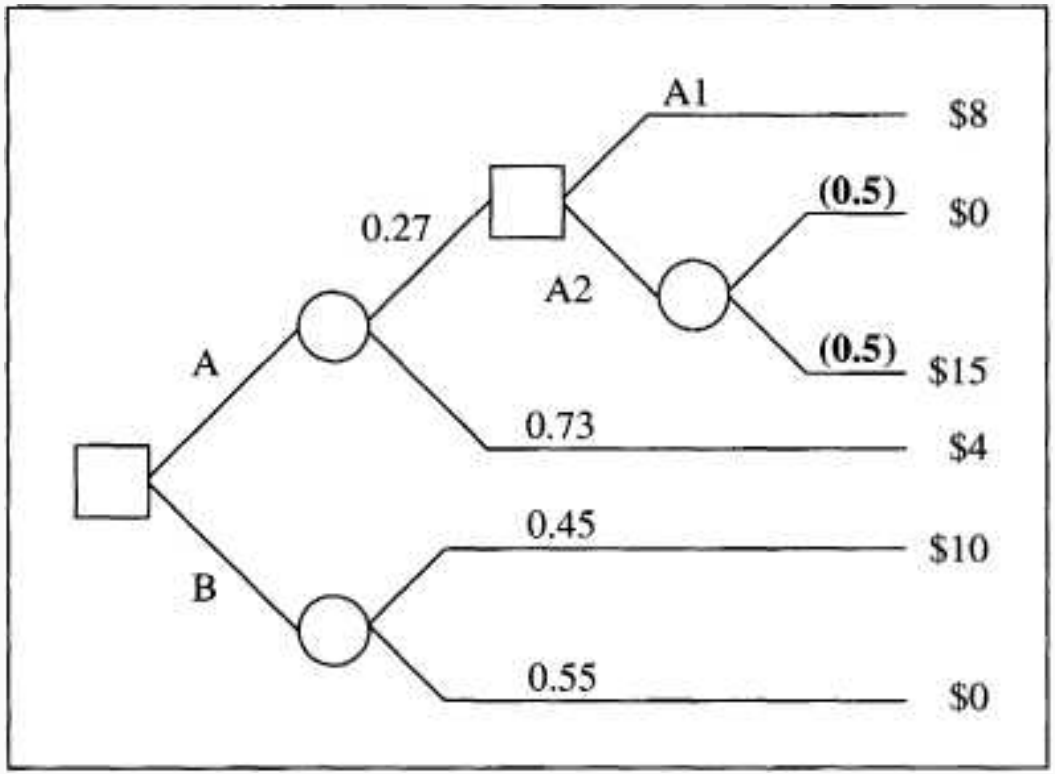


$$EV(ForestJob) = 0.10(88.6)+0.25(84.6)+0.40(72.6)+0.20(58.6)+0.05(48.6) = 73.2$$

$$EV(InTownJob) = 0.35(84) + 0.50(48) + 0.15(24) = 57$$

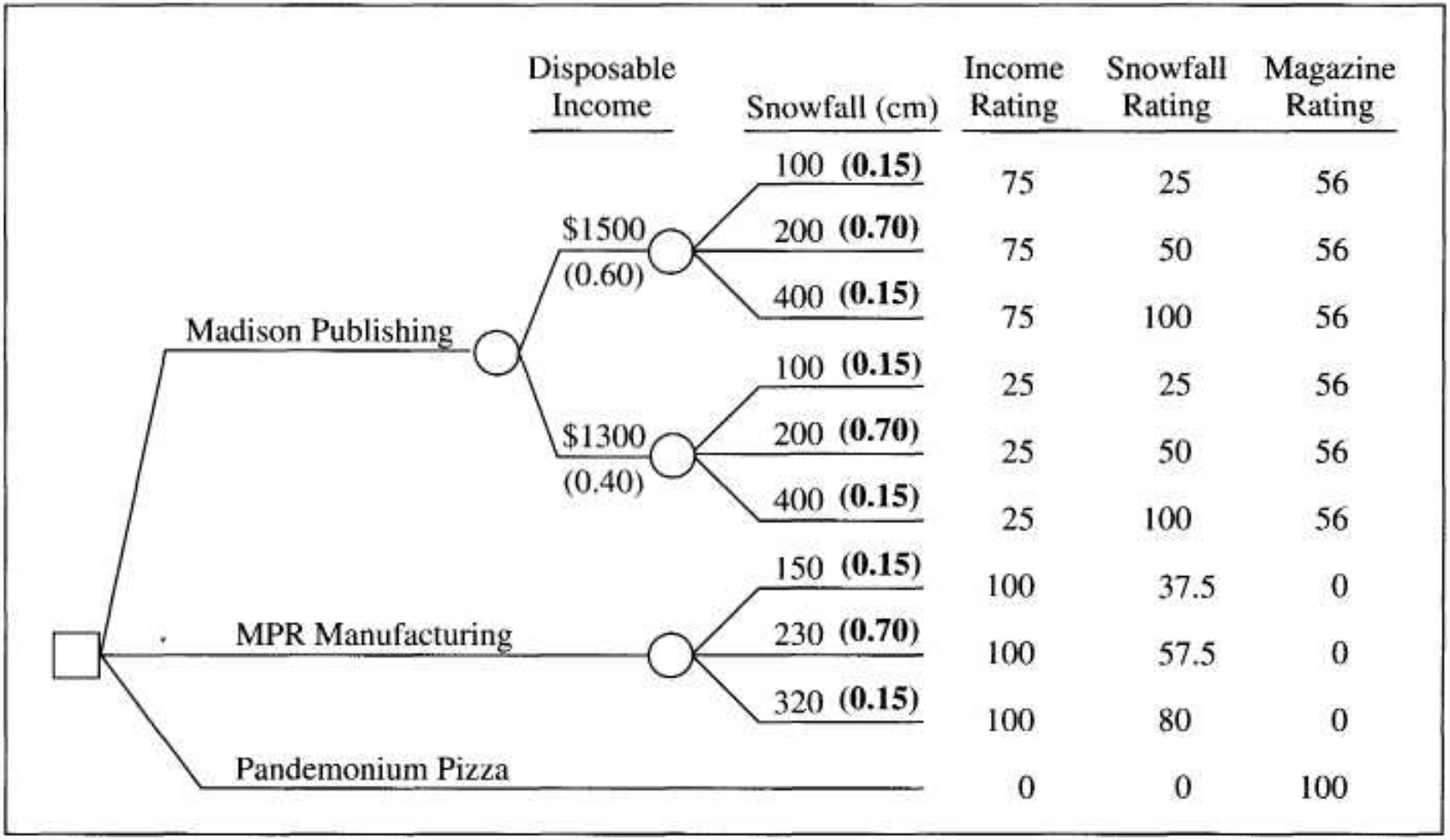
Quiz

1. Solve the decision tree below with EMV
2. Build the risk profiles for all possible decision paths



Quiz

Given the decision tree below, solve the multiple-objectives problem. Assign the trade-off importance according to your judgment.



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- in decision analysis:
  - after a model has been chosen ->
  - *if we make a slight change to the model parameters, does the optimal decision change?*
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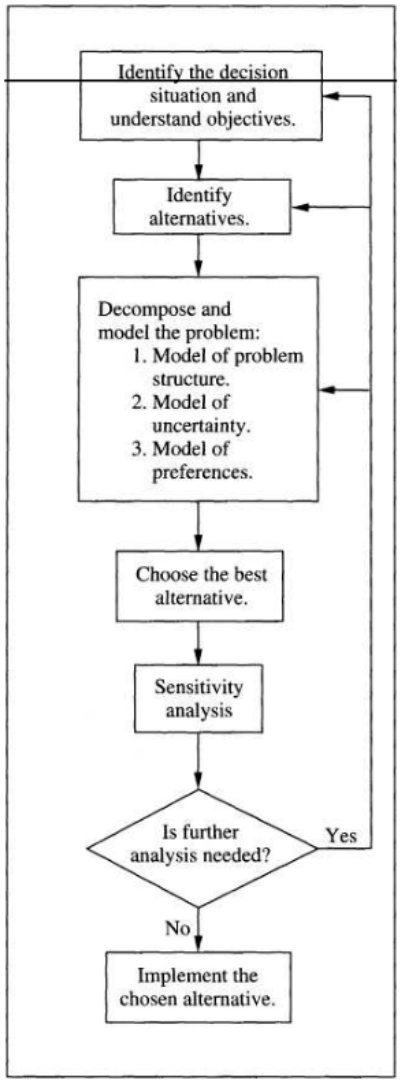
Let's look back at the decision analysis process

The Decision Analysis Process

Figure 1.1
A decision-analysis process flowchart.



1. Problem identification
   - sometimes we treat the wrong problem
   - type III problem (right answer to the wrong question)
   - "I have a headache"
     - headache only?
     - symptom of another illness?

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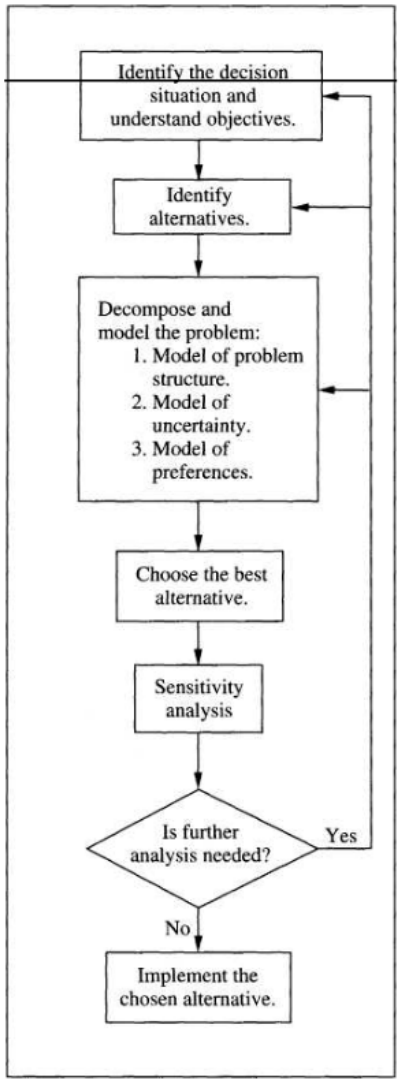

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2. Identify objectives and alternatives
   - minimizing costs?
   - maximizing profit?
   - minimizing risk?
     - Money loss? Health?
   - consideration of many aspects (objectives) leads to unforseen alternatives

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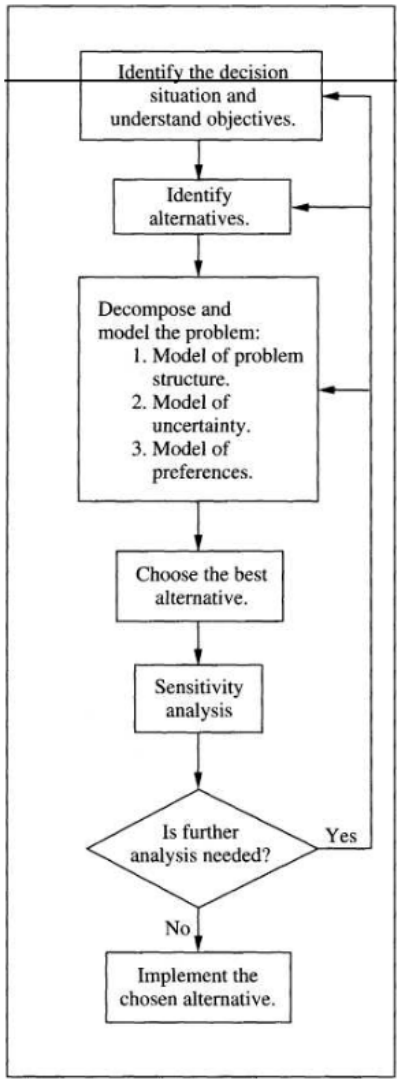

3. Modeling and Solutions

- divide and conquer = decompose problems in order to understand their structures and measure uncertainty and value
  - Modeling decisions (structuring)
    - influence diagrams
    - decision trees
  - Modeling uncertainty
    - probability
  - Models of outcome value (preferences)
    - utility functions

Sensitivity analysis

- steps 1 and 2 should be addressed separately
- step 3 is addressed using sensitivity analysis tools
  - one-way SA
    - tornado diagrams
  - two-way SA
    - 2-variable graphs

Example: Eagle Airlines

Dick Carothers, president of Eagle Airlines, had been considering expanding his operation, and now the opportunity was available. An acquaintance had put him in contact with the president of a small airline in the Midwest that was selling an airplane. Many aspects of the situation needed to be thought about, however, and Carothers was having a hard time sorting them out. Eagle Airlines owned and operated three twin-engine aircraft. With this equipment, Eagle provided both charter flights and scheduled commuter service among several communities in the eastern United States. Scheduled flights constituted approximately 50% of Eagle's flights, averaging only 90 minutes of flying time and a distance of some 300 miles. The remaining 50% of flights were chartered. The mixture of charter flights and short scheduled flights had proved profitable, and Carothers felt that he had found a niche for his company. He was aching to increase the level of service, especially in the area of charter flights, but this was impossible without more aircraft. A Piper Seneca was for sale at a price of $95,000, and Carothers figured that he could buy it for between $85,000 and $90,000. This twin-engine airplane had been maintained according to FA A regulations. In particular, the engines were almost new, with only 150 hours of operation since a major overhaul. Furthermore, having been used by another small commercial charter service, the Seneca contained all of the navigation and communication equipment that Eagle required. There were seats for five passengers and the pilot, plus room for baggage. Typical airspeed was approximately 175 nautical miles per hour (knots), or 200 statute miles per hour (mph). Operating cost was approximately $245 per hour, including fuel, maintenance, and pilot salary. Annual fixed costs included insurance ($20,000) and finance charges. Carothers figured that he would have to borrow some 40% of the money required, and he knew that the interest rate would be two percentage points above the prime rate (currently 9.5% but subject to change). Based on his experience at Eagle, Carothers knew that he could arrange charters for $300 to $350 per hour or charge a rate of approximately $100 per person per hour on scheduled flights. He could expect on average that the scheduled flights would be half full. He hoped to be able to fly the plane for up to 1000 hours per year, but realized that 800 might be more realistic. In the past his business had been approximately 50% charter flights, but he wanted to increase that percentage if possible. The owner of the Seneca has told Carothers that he would either sell the airplane outright or sell Carothers an option to purchase it within a year at a specified price. (The current owner would continue to operate the plane during the year.) Although the two had not agreed on a price for the option, the discussions had led Carothers to believe that the option would cost between $2500 and $4000. Of course, he could always invest his cash in the money market and expect to earn about 8%. As Carothers pondered this information, he realized that many of the numbers he was using were estimates. Furthermore, some were within his control (for example, the amount financed and prices charged) while others, such as the cost of insurance or the operating cost, were not. How much difference did these numbers make? What about the option? Was it worth considering? Last, but not least, did he really want to expand the fleet? Or was there something else that he should consider?

Simplified

- Dick Carothers (Eagle Airlines) wants to expand his operations (more charter flights -> more airplanes)
  - charter flights
  - short scheduled flights
- small airline is selling an aircraft 95k, he hopes for 85k-90k
- annual fixed costs (insurance, finance charges)
- borrow 40% of money at cca 9.5%+2%
- expected half-full flights, but wanted to increase if possible
- sell immediately or sell an option to buy within a year at a price to be defined (option would be cca 2.5k-4k)
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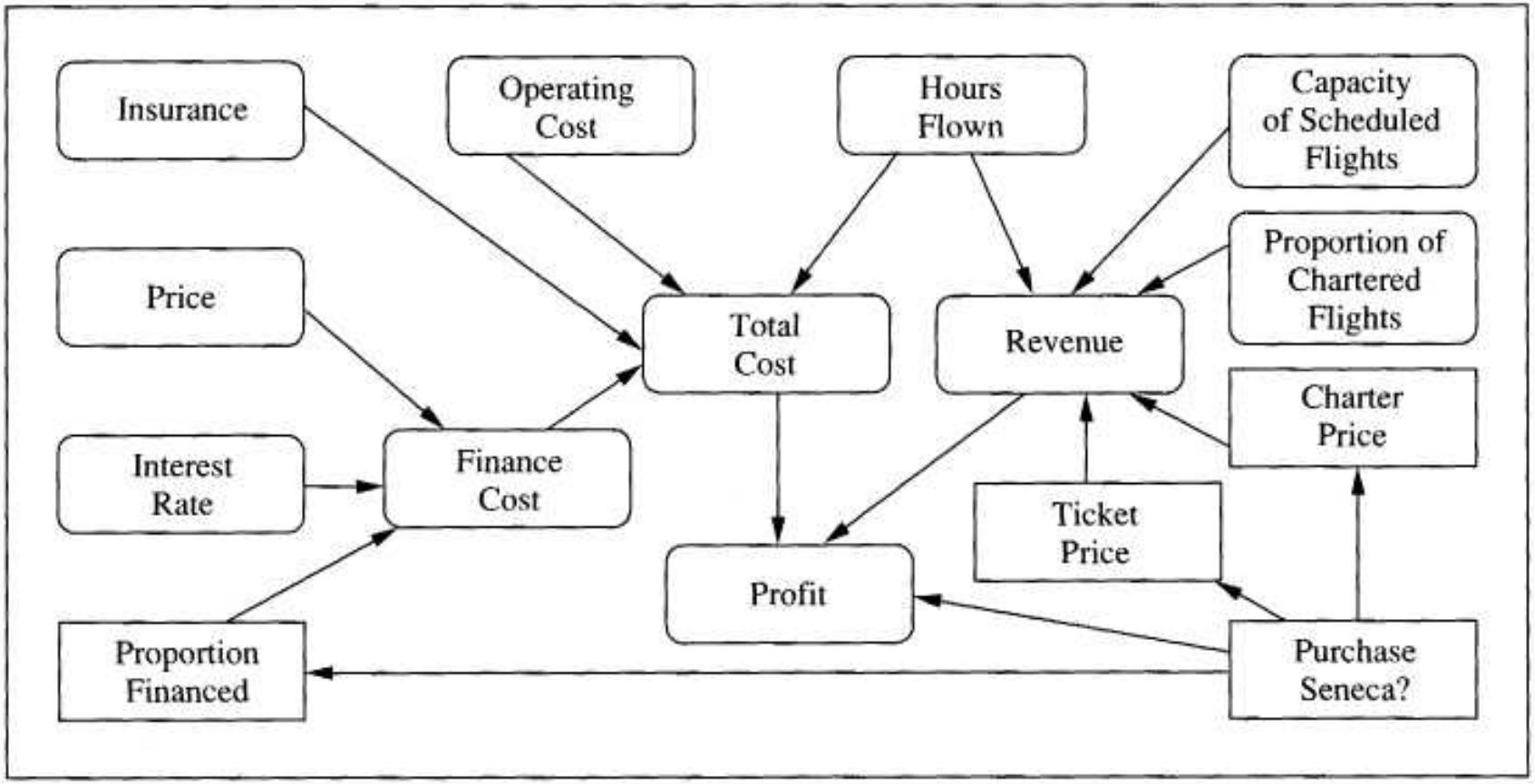
- most of the above numbers are estimates
- some are under control, some are not
- how much influence do these estimates have on the outcome
- what about alternative decisions (the option)
- does he really want to expand the fleet?

Influence Diagram



- rounded: constants, intermediate calculations, consequences
- rectangles: decisions

Input Variables and Possible Ranges

Variable	Base Value	Lower Bound	Upper Bound
Hours Flown	800	500	1000
Charter Price/Hour	$325	$300	$350
Ticket Price/Hour	$100	$95	$108
Capacity of Scheduled Flights	50%	40%	60%
Proportion of Chartered Flights	0.50	0.45	0.70
Operating Cost/Hour	$245	$230	$260
Insurance	$20,000	$18,000	$25,000
Proportion Financed	0.40	0.30	0.50
Interest Rate	11,5%	10.5%	13%
Purchase Price	$87,500	$85,000	$90,000

- annual profit = total annual revenue - total annual cost
- annual profit = 230.000 - 220.025 = 9975
- annual return of investment (ROI) = 19% (of 52.500 = 60% of purchase price)

Formula for annual revenue

Total Revenue = Revenue from Charters + Revenue from Scheduled Flights

= (Charter Proportion × Hours Flown × Charter Price) + [(1 — Charter Proportion) × Hours Flown × Ticket Price × Number of Passenger Seats × Capacity of Scheduled Flights]

= (0.5 × 800 × $325) + (0.5 × 800 × $100 × 5 × 0.5)

= $230,000 Total Cost = (Hours Flown X Operating Cost) + Insurance + Finance Cost

= (Hours Flown × Operating Cost) + Insurance + (Price × Proportion Financed × Interest Rate)

= (800 × $245) + $20,000 + ($87,500 × 0.4 × 11.5%)

= $220,025

One-way Sensitivity Analysis

- Which variables make a difference in terms of the decision at hand?
  - does changing the ticket price affect the decision?
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- a simple approach is **one-way sensitivity analysis**

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- choose one variable (e.g. number of flying hours)
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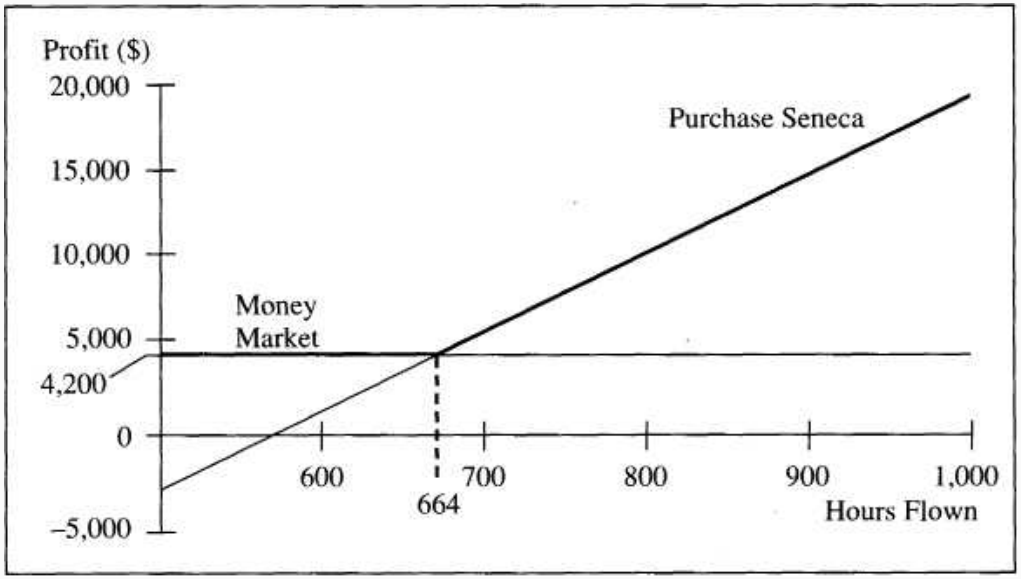
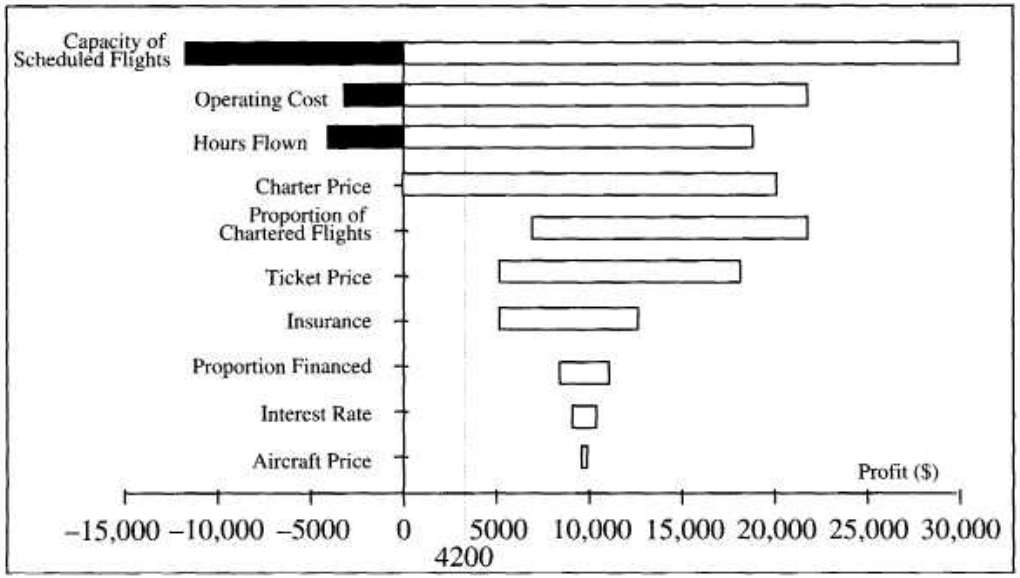


- the MoneyMarket horizontal line = is the revenue if they put the investment on the market (8% of 52.500)
- intersection:
  - left: better invest money on the market
  - right: better buy aircraft

Tornado diagrams

- do one-way sensitivity analysis with **multiple variables**
- we take (observe) a variable:
  - fix the others to base values
  - calculate the value of the outcome for the range of the observed variable

Example Tornado diagram



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Quiz

Calculate a tornado diagram for a one-way sensitivity analysis in the case of the decision of buying or renting a flat for yourself. Choose 4 variables, compose a formula for assessing the outcome.

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variables	Base	Low	High
rent	400	300	800
months	240	120	360
price	100000	80000	150000
monthlyInterest	50	30	80

$$moneySpentWithBuy = price + months \cdot monthlyInterest$$

$$moneySpentWithRent = rent \cdot months$$

Variable	Base	Min	Max	min(rent-buy)	max(rent-buy)
Rent	€ 400,00	€ 300,00	€ 800,00	€ -40.000,00	€ 80.000,00
Months	240	120	360	€ -58.000,00	€ 26.000,00
Price	€ 100.000,00	€ 80.000,00	€ 150.000,00	€ 4.000,00	€ -66.000,00
Monthly interest	€ 50,00	€ 30,00	€ 80,00	€ -11.200,00	€ -23.200,00

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- for all the variables, except the observed two, we insert the base values
- then we solve the inequality

$$Capacity < 0.004 \cdot OperatingCost - 0.509$$

Two-way Sensitivity Analysis

$$Capacity < 0.004 \cdot OperatingCost - 0.509$$

- we insert two extremes of one variable and calculate the other to find the border
- $OperatingCost \in [230, 260]$

$$Capacity(230) < 0.004 \cdot 230 - 0.509$$

$$Capacity(230) < 0.411$$

$$Capacity(260) < 0.004 \cdot 260 - 0.509$$

$$Capacity(260) < 0.531$$

Two-way Sensitivity Analysis

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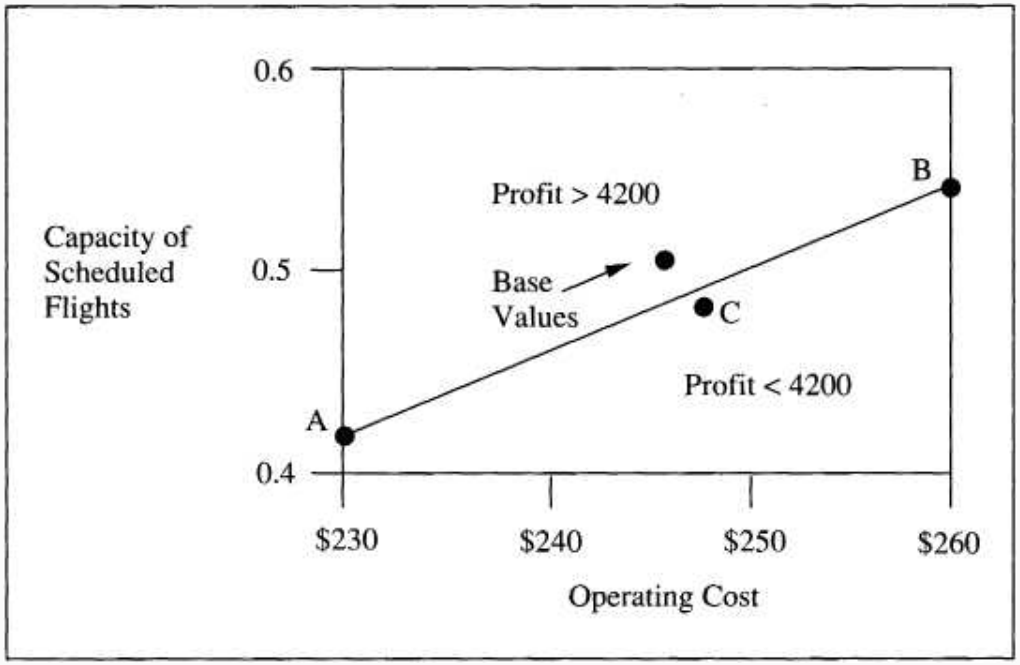

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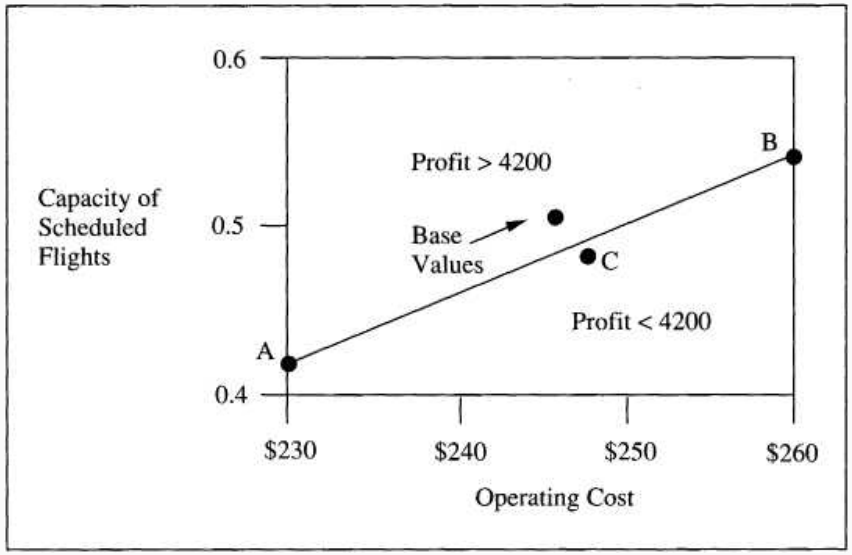

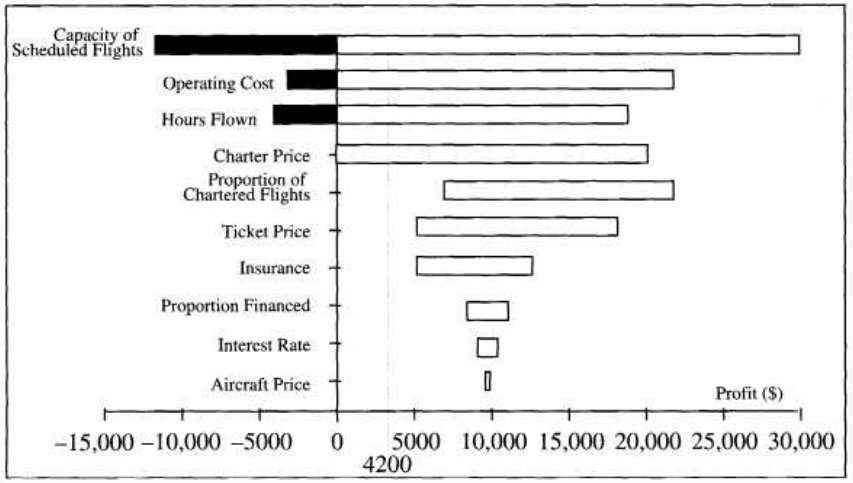

- point C in the two-way graph:
  - $OperatingCost = 248$ (profit in the tornado single diagram)
  - $Capacity = 0.48$ (profit in the tornado single diagram)

Sensitivity to Probabilities

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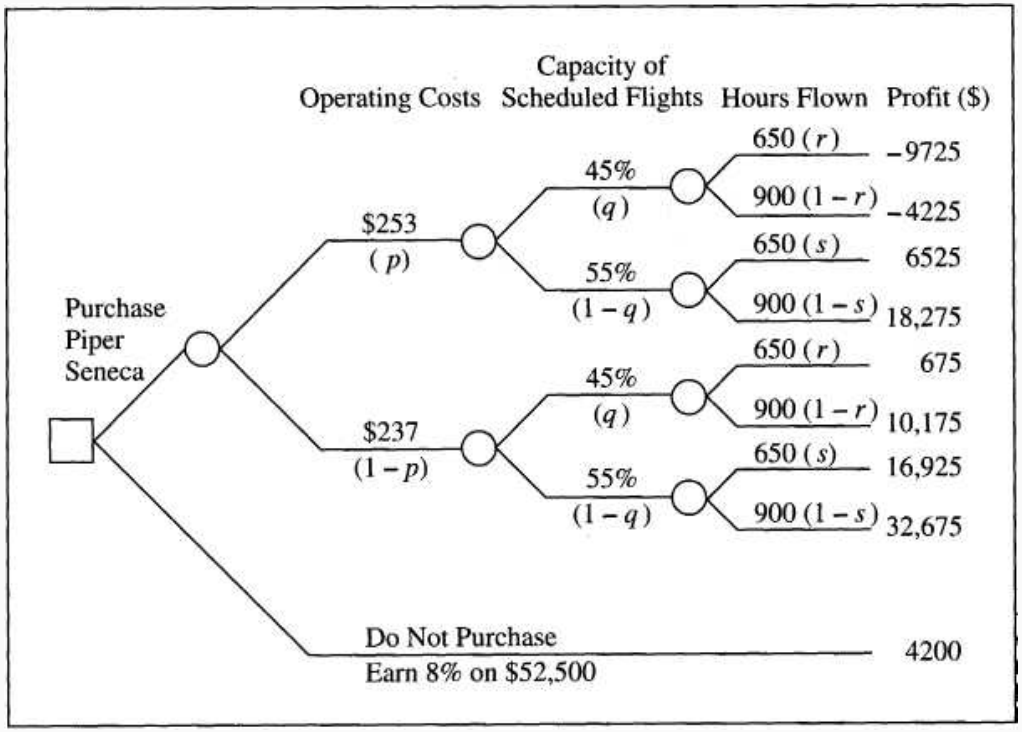
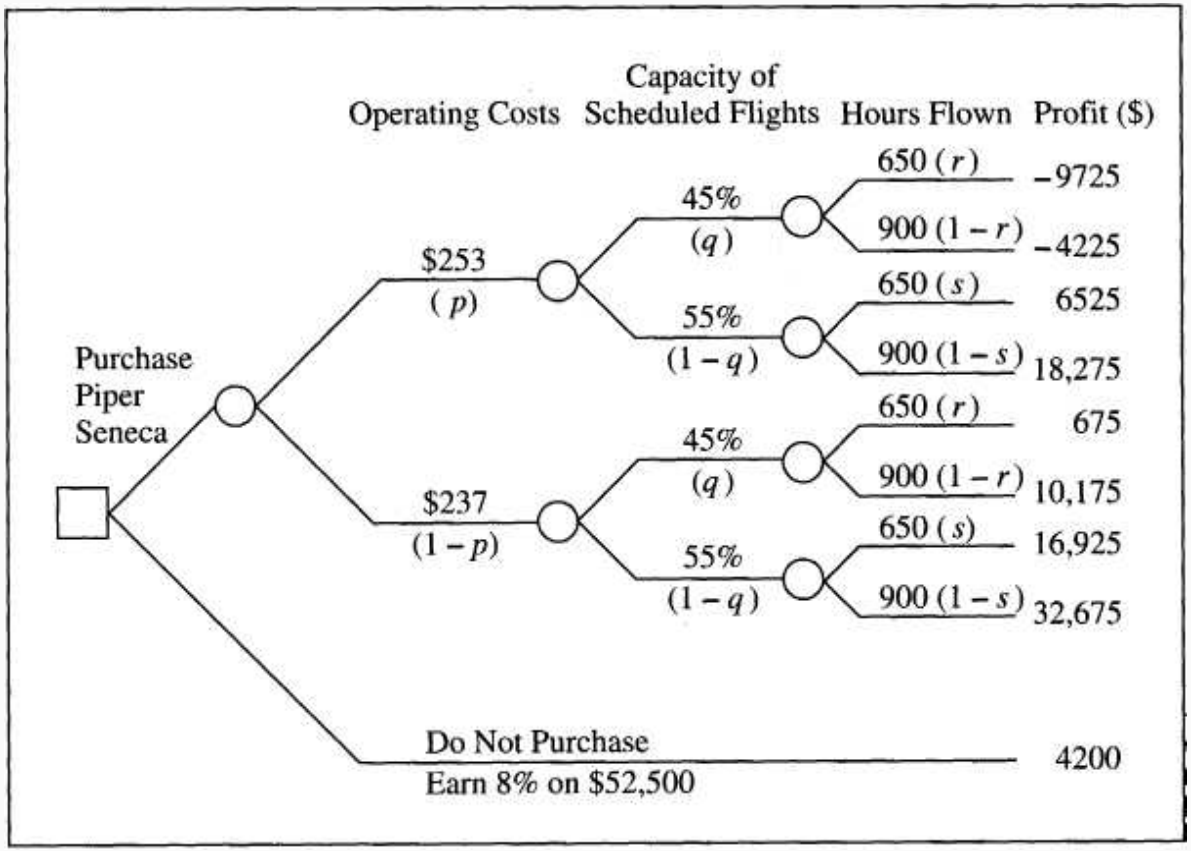
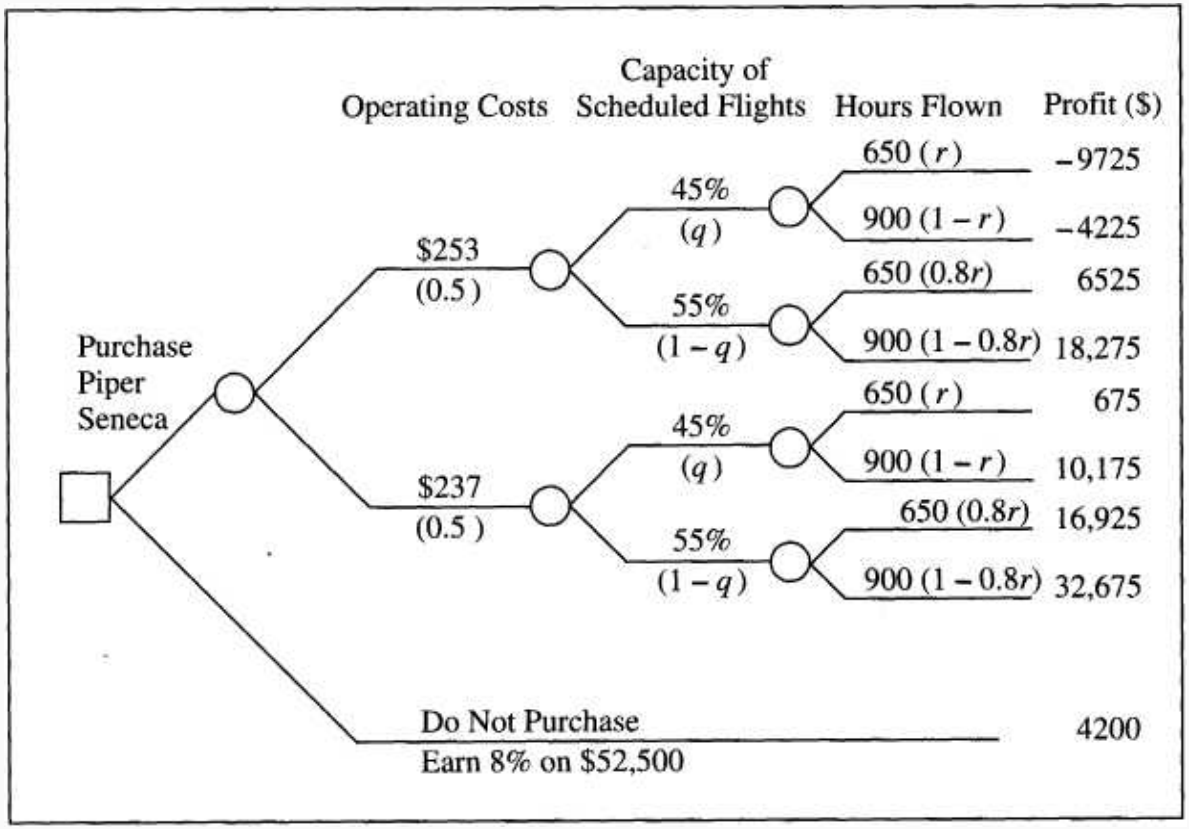


Sensitivity to Probabilities



- it is up to our judgment how to assign the probabilities p, q, r and s
- example:
  - $p = 0.5$
  - $s = 0.8 \cdot r$,

Sensitivity to Probabilities



- modified decision tree with $p = 0.5$ and $s = 0.8 \cdot r$
- having two unknown variables (this time the probabilities q and r and not their respective values) we can do the two-way sensitivity analysis

Sensitivity to Probabilities

- this time we base our two-way sensitivity analysis on the **EMV** :

$$EMV(Purchase) > 4200$$

$$EMV(Purchase) = 0.5\{q[-9725r - 4225(1-r)] + (1-q)[6525(0.8r) + 18275(1-0.8r)]\} +$$

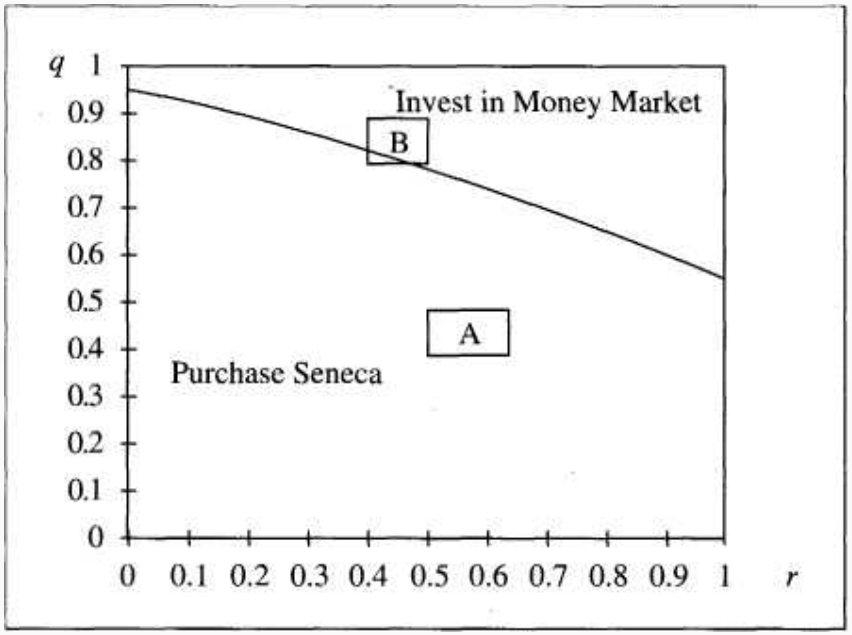
$$+0.5q[675r + 10175(1-r)] + (1-q)[16925(0.8r) + 32675(1-0.8r)]$$

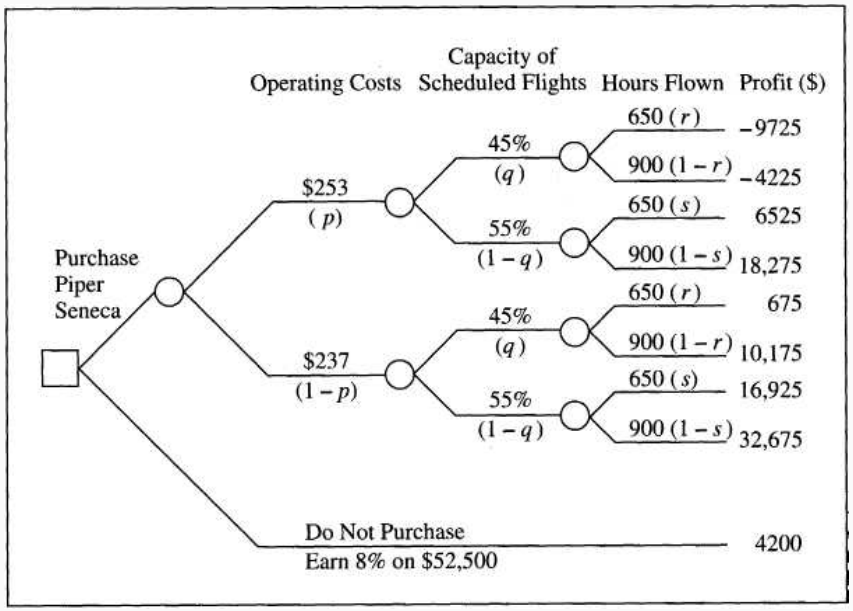
$$EMV(Purchase) = q(3500r - 22500) - 11000r + 25475$$

$$\frac{21275 - 11000r}{22500 - 3500r} > q$$

Sensitivity to Probabilities

$$\frac{21275 - 11000r}{22500 - 3500r} > q$$

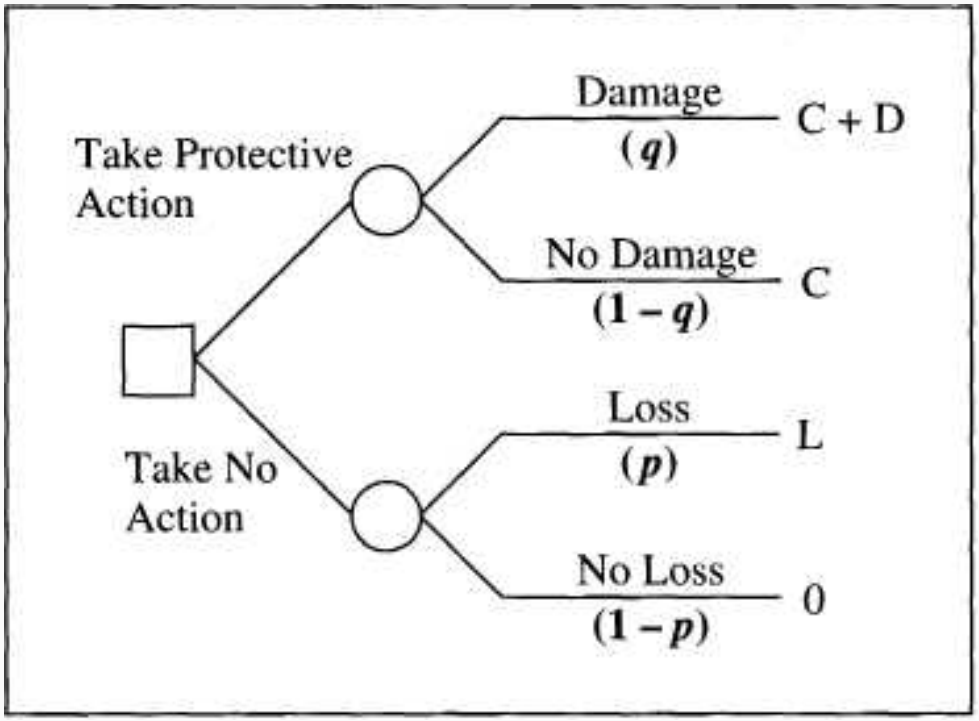




- high p and q -> pessimistic assessment
- this kind of graph tells whether the decision **is sensitive to the uncertainty** (and not to the respective values)

Quiz

Orchard. A protective action that may be taken may not provide perfect protection. Suppose that, even with protective action, damage D will be sustained with probability q. Thus, the decision tree appears as below. Explain how sensitivity analysis could be used to determine whether it is important to include the upper chance node with probability q and damage D. Use the appropriate sensitivity analysis tools.



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